

## REPORT No. 26

### THE VARIATION OF YAWING MOMENT DUE TO ROLLING

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Contribution, 1919, No. 1, from the Rogers Laboratory of Physics Massachusetts Institute of Technology.

#### PREVIOUS WORK

The aerodynamical constants of an airplane necessary for the discussion of stability are partly observed and partly calculated. Among the calculated coefficients is  $N_p$ , which is the variation of yawing moment due to rolling. In the Technical Report of the Advisory Committee for Aeronautics (London), 1912-13, Reports and Memoranda No. 77, being an Investigation into the Stability of an Aeroplane, etc., by L. Bairstow and others, on page 157 it is stated that "for the wings it will be seen that whilst  $L_p$  is proportional to the slope of the lift curve,  $N_p$  is proportional to the slope of the drag curve. Hence  $N_p$  will be one-tenth of  $L_p$  at angles slightly above that giving maximum lift/drag, and may become zero, or even slightly negative, in a machine flying at about the angle of minimum drag. The effect of the rudder and body will be appreciable in most machines.  $N_p$  will be variable between the limits 0 and 40."

In a contribution on "Dynamical stability of aeroplanes," by Jerome C. Hunsaker, Smithsonian Miscellaneous Collections (Washington), vol. 62, No. 5, June, 1916, pages 55-57, the calculation of  $N_p$  is carried out in detail along the lines laid down in the above quotation and values for the machine under discussion (a design by Capt. V. E. Clark, United States Army) run from 0 to 57.

An essential argument in these derivations of a value for  $N_p$  is that the roll produces an increased line of attack on an elementary area of the right wing and a diminished angle of attack on the corresponding element of the left wing; and that, consequently, the variation of the yawing moment should be calculated from the slope of the drag curve. Now, as a matter of fact, this would be certainly correct in case the wing were a flat plane to which the resultant pressure remained always normal, independently of the angle of attack; but it is by no means certain that the argument is valid in the case of cambered wings, where a change in the angle of attack produces a change in the direction of the resultant force as well as a change in its magnitude. It would seem to be more likely to be correct to take account of the change in direction of the resultant force, as is the case in the calculation of  $X_w$ , the variation in the  $X$  force due to vertical velocity. In the case of the flat plane  $X_w$  is a negative quantity, owing to the diminution of the resultant pressure (its direction remaining invariant) when the angle of attack is diminished; but the change in direction of the resultant force on the cambered wing is so much more important than the diminution in its magnitude (owing to the large value of the lift relative to the drag) that the value of  $X_w$  for the airplane becomes actually positive instead of negative.

#### A NEW CALCULATION

It would seem that in calculating  $N_p$  the change in the  $X$  force (not in drag alone) should be used. It would be possible to make the calculation on this basis in the following manner:

Let  $dy$  be an element of length along the wing, and  $S$  its span. If  $m$  is the mass of the airplane in slugs,  $mX_w w$  is the variation in the actual  $X$  force, due to  $w$ ; and for the element  $dy$

the proportional part would be  $mX_w w dy/S$ . The moment of this change is the change itself multiplied by its arm  $y$ , viz.,  $mX_w wy dy/S$ ; and the total value of the variation in the moment

$$mN_p p = \int_{-S/2}^{+S/2} mX_w wy dy/S.$$

The value of  $w$  is negative, and equal to  $-yp$ . On substituting this value, we have

$$N_p = \int_{-S/2}^{+S/2} -X_w y^2 dy/S = -X_w S^3/12.$$

When the values for the Clark design are substituted, in which  $S=40$ , the following results are found:

$U$	$= -112.5$	$-78.2$	$-65.3$	$-54$
$i$	$= 0^\circ$	$3^\circ$	$6^\circ$	$12^\circ$
$X_w$	$= +.356$	$+.249$	$+.245$	$0$
$N_p$	$= -47.5$	$-33.2$	$-32.7$	$0$

whereas, according to Hunsaker, the values of  $N_p$  are

$$N_p = 0 \qquad \qquad \qquad +33.5 \quad +57.0$$

It will be observed that  $N_p$  by this calculation is negative, instead of positive, and that the numerical values are large for small angles of attack, small for large angles. Not only the sign of  $N_p$  is changed, but the general trend of numerical values is reversed.

#### EFFECT ON STABILITY

Fortunately the value of  $N_p$  is not of very serious moment in the discussion of stability. The expression in which it is most important is the approximate form of the damping in the type of motion which Hunsaker (loc. cit., p. 71) calls the Dutch roll, and which corresponds to the quadratic factor of the biquadratic that governs the lateral motion, namely:

$$D^2 + \left( \frac{C_2}{B_2} - \frac{E_2}{D_2} \right) D + \frac{B_2 D_2}{B_2^2 - A_2 C_2} = 0.$$

$$\frac{C_2}{B_2} - \frac{E_2}{D_2} = \frac{L_r}{K C^2} \left( \frac{N_p}{L_p} - \frac{N_v}{L_v} \right).$$

The value of  $L_p$  is negative. That of  $N_v$  is also negative, and those of  $L_v$  and  $L_r$  are positive. If, therefore,  $N_p$  be positive, as found by Bairstow and Hunsaker, the expression in the parenthesis is the difference of two quantities; and in order to insure stability, it is necessary that this difference be positive—that is, we must have  $N_v/L_v$  greater numerically than  $N_p/L_p$ . Now,  $N_v$  and  $L_v$  occur in the expression which determines spiral stability or instability, and the ratio  $-N_v/L_v$  is desired small for spiral stability, whereas it is desired large for stability in the Dutch roll. Thus, there arises the necessity for a very fine compromise in the relative magnitudes of  $N_v$ ,  $L_v$ ,  $N_p$ ,  $L_p$ , in order that the machine may be stable both spirally and in the Dutch roll.

If, however, the value of  $N_p$  be negative, as is indicated by my calculation above, both terms in the parenthesis ( $N_p/L_p - N_v/L_v$ ) are positive for most attitudes of flight which have been examined, and the machine is stable in the Dutch roll without the necessity for any fine adjustment as compared with the spiral case. This should be a matter of some relief to the conscientious designer critical of the dynamic stability of his design.

#### CHECK ON THE CALCULATION

In so far as my argument for the calculation of  $N_p$  is just, a similar argument could be given for obtaining a calculated value of  $L_p$  in terms of  $Z_w$ , with the result:

$$L_p = Z_w S^3/12.$$

The values of  $L_p$  as calculated by this formula would be:

$U$	$= -112.5$	$- 78.2$	$- 65.3$	$- 54.0$
$i$	$= 0^\circ$	$3^\circ$	$6^\circ$	$12^\circ$
$Z_w$	$= - 5.62$	$- 3.77$	$- 2.92$	$- 1.0$
$L_p$	$= -749$	$-493$	$-389$	$-133$

whereas the values given by Hunsaker are

$$L_p = -631 \qquad -319 \qquad -224$$

The values of  $L_p$  are obtained rather easily by measuring the damping of the model when oscillating about the  $X$  axis, and, consequently, the measured values should be fairly trustworthy. The only two cases in the table in which a comparison can fairly be made are those in the first and last columns corresponding to highest and lowest speeds; for the integer in the third column is not an experimental value, but one obtained by interpolation. At the highest speed the calculated value of  $L_p$  is nearly 20 per cent too high; whereas at the lowest speed it is distinctly too low.

It would not be surprising if a calculated value based on  $Z_w$  should be too high, for the experimental method of determining  $Z_w$  is to compare the  $Z$  forces for the model when set at different angles of pitch relative to the fixed direction of the air current. Now, it is a common observation in our wind tunnel at the Massachusetts Institute of Technology that when the orientation of the model relative to the wind is quickly changed, a very considerable time elapses before the forces reach their steady value. It appears as though it took a reasonable amount of time for the stream lines in the fluid to change from one steady direction to another.

If this be so, it would be impossible for the stream lines to accommodate themselves to the oscillatory motion in the experimental determination of  $L_p$  as fully as they accommodate themselves to the changed orientation in the experimental determination of  $Z_w$ . The result would be that the effective value of  $Z_w$ , which should be used in the calculation of  $L_p$ , might be a considerable amount below the measured value of  $Z_w$ . No such explanation could be given for the discrepancy between the values as calculated and observed of  $L_p$  at the lowest speed, for the direction of the change is reversed. There is, however, the possibility that the value of  $Z_w$  as calculated from the experiments should be considerably in error, because this value must be obtained either by an interpolation in a table of values, or by estimating the slope of an experimentally determined curve, and either of these processes is one in which it is difficult to obtain accuracy, because the experimental errors or an error of judgment in fairing a curve are extremely effective in vitiating the value obtained for the rate of change of the ordinate in the vicinity of any particular point on the curve.

A comparison may be made for the Curtiss J. N. 2., from Hunsaker's data (loc. cit., p. 78) and these reports, First Annual Report, 1915, pp. 47-49:

$U$	$i$	$Z_w$	$L_p(\text{obs.})$	$L_p(\text{calc.})$	$X_w$	$N_p(\text{new})$	$N_p(\text{old})$
-115.5	$1^\circ$	-3.95	-314	-427	.162	-17.2	0
- 63.8	$15^\circ.5$	-.673	- 78	- 73	-.292	+31.5	+37.7

Here the calculated  $L_p$  at low speed checks very well with that observed, but is again considerably too high at high speeds. The reversal of sign of  $X_w$  for this machine has brought the two values of  $N_p$  for low speed near together.

#### EXPERIMENTAL CHECK

In the British report (1912-13), Reports and Memoranda No. 78, being the Experimental Determination of Rotary Coefficients, by L. Bairstow, etc., on pages 177-179 there is outlined an experimental method of measuring the value of  $L_p$ , the variation of the rolling moment due to yawing by a somewhat intricate experimental procedure, based on the theory of forced oscillations. In a similar manner, the variation of yawing moment due to rolling,  $N_p$ , could

be determined. It seems, however, that the measurement of the amplitude of the forced oscillation would determine only the numerical magnitude, and not the sign of  $L_r$  or  $N_p$  as the case might be; and that for the experimental determination of the sign it will be necessary to observe the phase difference between the forced oscillation and the periodic applied force. This should not be a difficult thing to observe, but it is quite possible that if one knew, or thought he knew, in advance what the sign of the result should be, he might overlook the matter of checking the sign by an observation on the phase difference of the two motions.

It is probable that before any great dependence can be put upon the calculated value of  $N_p$  or similar aerodynamic coefficients, an extended comparison of calculated values with experimentally determined ones will be necessary; and I have not offered the above discussion so much for the purpose of attempting a definitive determination of the value of  $N_p$  as for the purpose of finding out the possibility of making a calculation which would seem to be just as reasonable if not more reasonable than those before given; and which does, as a matter of fact, lead to a value of  $N_p$  of negative sign instead of one with a positive sign. The important thing for the discussion of stability is not so much the numerical value of  $N_p$ , unless  $N_p$  be positive, as the assurance that  $N_p$  is negative, if, indeed, it be negative.—(*Extract from lectures given at the Massachusetts Institute of Technology to a special course in aeronautical engineering for Army and Navy officers, May to September, 1918.*)

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NOTE.—In a book on Aeronautics, by Cowley and Levy, which has just come to hand, there is found on page 261 a table in which the value of  $N_p$  (in a notation different from that of Bairstow and Hunsaker) is negative; but the details of the calculation which lead to the value are not given, so that it can not be determined whether or not the negative value is intended or is a typographical error.